

Financially Constrained Fluctuations in an Evolving Network Economy

Domenico Delli Gatti ^a

Mauro Gallegati ^b

Bruce Greenwald ^c

Alberto Russo ^b

Joseph E. Stiglitz ^c

^a Università Cattolica, Milano, Italy

^b Università Politecnica delle Marche, Ancona, Italy

^c Columbia University, New York, USA

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Introduction

- Motivation:

- We study the properties of a **credit-network economy** characterized by credit relationships connecting downstream and upstream firm (**trade credit**) and firms and banks (**bank credit**).
- It is straightforward to think of *agents* as **nodes** and of *debt contracts* as **links**
- The **network topology** changes over time due to an endogenous process of **partner selection** in an **imperfect information** decisional context.
- The bankruptcy of one agent can bring about the bankruptcy of one or more other agents possibly leading to **avalanches of bankruptcies**.
- We investigate the interplay between **network evolution** and **business fluctuations** (bankruptcy propagation)
- “The **high rate of bankruptcy** is a cause of the **high interest rate** as much as a consequence of it” (Stiglitz and Greenwald, 2003: 145)
 - Agents' defaults → bad loans → deterioration of lenders' financial conditions → credit restriction (increase of the interest rate)
 - credit restriction (increase of the interest rate) → deterioration of borrowers' financial conditions → agents' defaults...

Introduction

- Related literature:

- Financial contagion in the **interbank market**: Allen and Gale (2000), Freixas et al. (2000), Furfine (2003), Boss et al. (2004), Iori et al., (2006), Nier et al. (2007) → interbank lending, liquidity management, network structure and financial crises.
- **Credit interlinkages**: Stiglitz and Greenwald (2003, Ch. 7) → a “circle” of connected firms (trade credit) linked to a bank (bank credit).
- Delli Gatti, Gallegati, Greenwald, Russo, Stiglitz (2006): business fluctuations (and bankruptcy propagation) in a three-sector economy (downstream firms, upstream firms and banks); **static network**
- The specific contribution of the present work is the introduction of a mechanism for the **endogenous evolution of the network**

The environment

- **Multi-sector** network economy:
 - Downstream sector ($i = 1, 2, \dots, I$ firms)
 - Upstream sector ($j = 1, 2, \dots, J$ firms)
 - Banking sector ($z = 1, 2, \dots, Z$ banks)
- **Discrete time** steps ($t = 1, 2, \dots, T$)
- **Two goods**: consumption and intermediate goods
- **Two inputs**: labour and intermediate goods
- **Downstream (D) firms** produce a perishable consumption good using labour and intermediate goods
- **Upstream (U) firms** produce intermediate goods “on demand” using only labour as input

The environment

- We rule out (by construction) the possibility of *avalanches of output* due to the mismatch of demand and supply of intermediate goods along the supply chain (Bak, Chen, Scheinkman and Woodford, 1993)
- The financial side of the economy is characterized by *two lending relationships*:
 - D and U firms obtain credit from banks
 - D firms buy intermediate goods from U firms by means of a commercial credit contract
- *Endogenous network formation* according to the *preferred-partner choice*:
 - In every period each D firm looks for the U firm with the lowest price of intermediate goods; at the same time each firm searches for the bank with the lowest interest rate
 - The number of potential partners an agent can check in each period is limited (*imperfect information*)

Firms

- The **core assumption** of the model is that the scale of activity of the i -th D firms at time t is an increasing concave function of its financial robustness, proxied by net worth (A_{it}):

$$Y_{it} = \phi A_{it}^\beta$$

where $\phi > 1$ and $0 < \beta < 1$ are parameters, uniform across D firms.

- Two rationales for the *financially constrained output function*:
 - A **simple rule of thumb** in a world in which
 - **Bounded rationality** prevents the elaboration of optimizing decision-making processes and
 - **Asymmetric information** between lenders and borrowers yields a financing hierarchy in which net worth ranks first.
 - Alternatively one can think of this equation as the solution of a firm's **optimization problem** (Greenwald and Stiglitz, 1993):
 - Max expected profits minus bankruptcy costs: increase of **financial fragility** (reduction of net worth) --> increase of **bankruptcy probability**

Firms

- Labour and intermediate goods **requirement functions** for D firms:
 - $N_{it} = \delta_d Y_{it}$ (demand for labour)
 - $Q_{it} = \gamma Y_{it}$ (demand for intermediate goods)where $\delta_d > 0$ and $\gamma > 0$.
- Final goods are sold at a **stochastic price** u_{it} , a random variable distributed in the interval $(0,2)$.
- In each period a U firm receives **orders** from a set of D firms ($_j$)
 - $_j$ depends on the price $p_{jt} = 1 + r_{jt}$, where r_{jt} is the interest rate on trade credit
 - The lower the price the higher the number of D firms placing order to j -th U firm
 - We assume that the interest rate depends on the firm's financial conditions:

where $_ > 0$.

$$r_{jt} = \alpha A_{jt}^{-\alpha}$$

Firms

- The **scale of production** of U firms is “demand constrained”: $Q_{jt} = \gamma \sum_{i \in \Phi_j} Y_{it}$
- Labour **requirement function** for U firms: $N_{jt} = \delta_u Q_{jt}$, where $\delta_u > 0$.
- **Financing hierarchy**: the financing gap (the difference between the firm's expenditures and internal finance) is filled by means of credit
 - U and D firms: wage bill – net worth
 - D firms: intermediate goods \rightarrow trade credit
- **Demand for credit**: $B_{xt} = W_{xt} - A_{xt}$
where $W_{xt} = wN_{xt}$ is the wage bill ($x=i$ for D firms, j for U firms)
- **Self-financed firms** (firms with a sufficient level of net worth to finance the wage bill) do not demand credit
- The **real wage** w is constant and uniform across firms

Banks

- In each period of time a set of (D and U) firms, denoted by \underline{z} , demands **credit** to the z -th bank (the lower the interest rate the larger the number of customers)
- The **interest rate** on the loan to the x -th borrower is:
$$r_{zt}^x = \sigma A_{zt}^{-\beta} + \theta (l_{xt})^\theta$$
where A_{zt} is the *net worth* of the bank and $l_{xt} = B_{xt}/A_{xt}$ is the *leverage ratio* of the x -th firm, σ and θ are positive parameters.
 - According to this rule:
 - Financially sound banks can extend credit at better conditions (they reduce the interest rate and attract more firms)
 - Banks penalizes financially fragile firms (the interest rate charged by the lender incorporates an *external finance premium*, increasing with leverage and therefore inversely related to the borrower's net worth)

Partner choice

- Each D firm has a (productive and credit) relationship with a U firm.
- At the beginning, links are established at random.
- In subsequent periods the network changes endogenously according to a **preferred-partner choice rule** (with noise):
 - with a (small) probability α , the D firm chooses a partner at random;
 - with probability $(1 - \alpha)$, it looks at the prices of a randomly selected number (M) of U firms
 - if the minimum observed price is lower than the price of the previous partner, it will switch to the new U firm
 - otherwise, it will stick to the previous partner
- The preferred-partner choice also applies to the relationships between firms (both D and U) and banks

Profits, net worth and bad debt

- The **profit** of *i*-th **D firm** is: $\pi_{it} = u_{it}Y_{it} - (1 + r_{zt}^i)B_{it} - (1 + r_{jt})Q_{it}$
- The **profit** of the *j*-th **U firm** is: $\pi_{jt} = (1 + r_{jt})Q_{jt} - (1 + r_{zt}^j)B_{jt}$
- The **profit** of the *z*-th **banks** is: $\pi_{zt} = \sum_{i \in I_z} (1 + r_{zt}^i)B_{it} + \sum_{j \in J_z} (1 + r_{zt}^j)B_{jt}$
- At the end of the period, the **net worth** of the *x*-th agent (x=i for D firms, j for U firms and z for banks) is: $A_{xt+1} = A_{xt} + \pi_{xt} - BD_{xt}$

where BD is **bad debt** (non-performing loans).

- In the case of **U firms**: $BD_{jt} = (1 + r_{jt}) \gamma \sum_{i \in \Phi_j^B} Y_{it}$

- In the case of **banks**: $BD_z = \sum_{i \in \Phi_z^B} (1 + r_{zt}^i)B_{it} + \sum_{j \in \Phi_z^B} (1 + r_{zt}^j)B_{jt}$

- The agent goes **bankrupt** if $A_{xt+1} < 0$.

Simulations

- **Agents:** $I = 500$ (D firms); $J = 250$ (U firms), and $Z = 100$ (banks).
- **Time span:** $T = 1000$.
- **Parameter setting:**
 - Financially constrained output of D firms: $\phi = 1.5$; $\underline{} = 0.8$;
 - Labour requirement of D and U firms: $\delta_d = 0.5$; $\delta_u = 1$;
 - Intermediate goods requirement of D firms: $\gamma = 0.5$;
 - Interest rate on trade credit: $\underline{} = 0.1$;
 - Interest rate on bank credit: $\underline{} = 0.1$; $\underline{} = 0.05$;
 - Real wage: $w = 1$;
 - Number of potential partners: $M = 5$; $N = 5$;
 - Probability of preferred-partner choice: $1 - \varepsilon = 0.99$.
- **Initial conditions:** new worth is set to 1 for all agents
- **Entry-exit process:**
 - One-to-one replacement: net worth of new entrants is drawn from a uniform distribution with support $(0,2)$

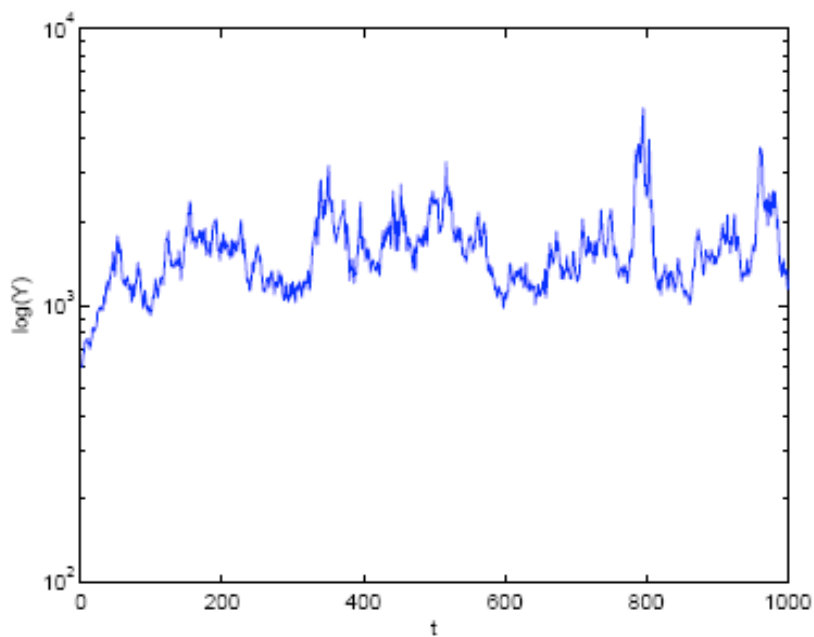


Figure 1: Aggregate Production

- **Aggregate production of D firms:** As expected in complex adaptive systems, **fluctuations are irregular** (amplitude and periodicity vary from period to period)
- **Aggregate production of U firms** follows the same dynamic pattern since U suppliers produce intermediate goods for D firms “on demand”.

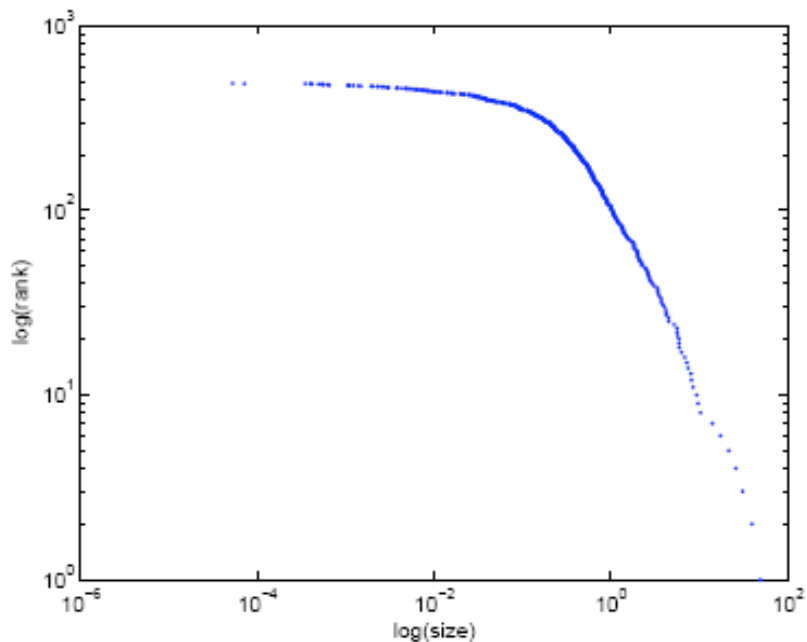
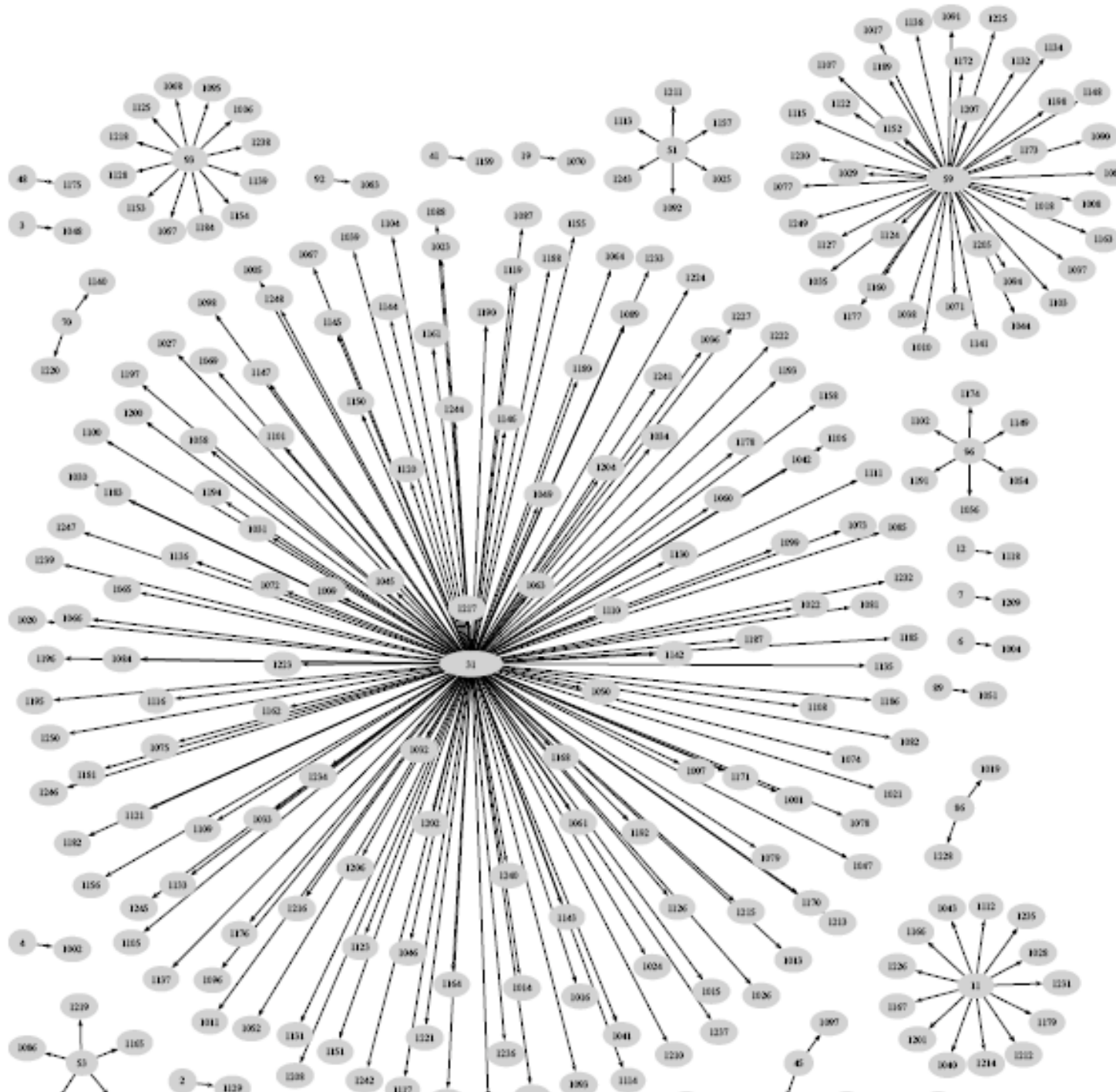


Figure 2: Firm size distribution (DS firms' net worth)

- Starting from **identical initial conditions** agents become rapidly heterogeneous
- **Firm size distribution** tends to a power law



**Network structure:
U firms vs. banks**

The number of links for each lender (U firm or bank) becomes asymmetric over time due to the preferred-partner choice governing interaction among borrowers and lenders

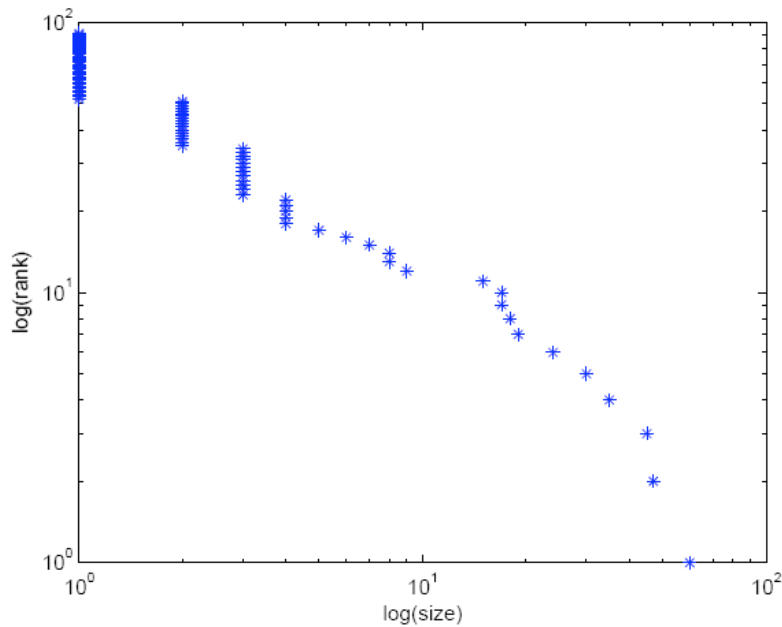


Figure 3: Degree distribution of the network: DS firms vs. US firms

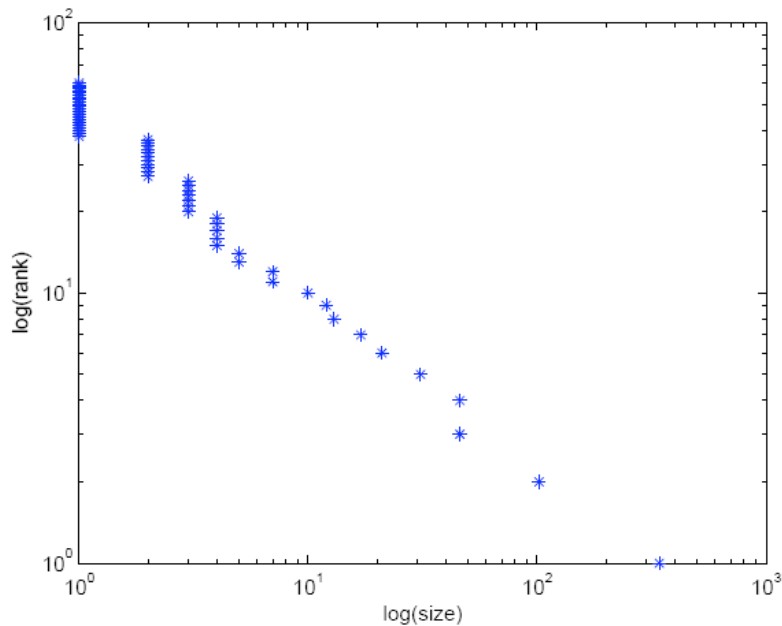
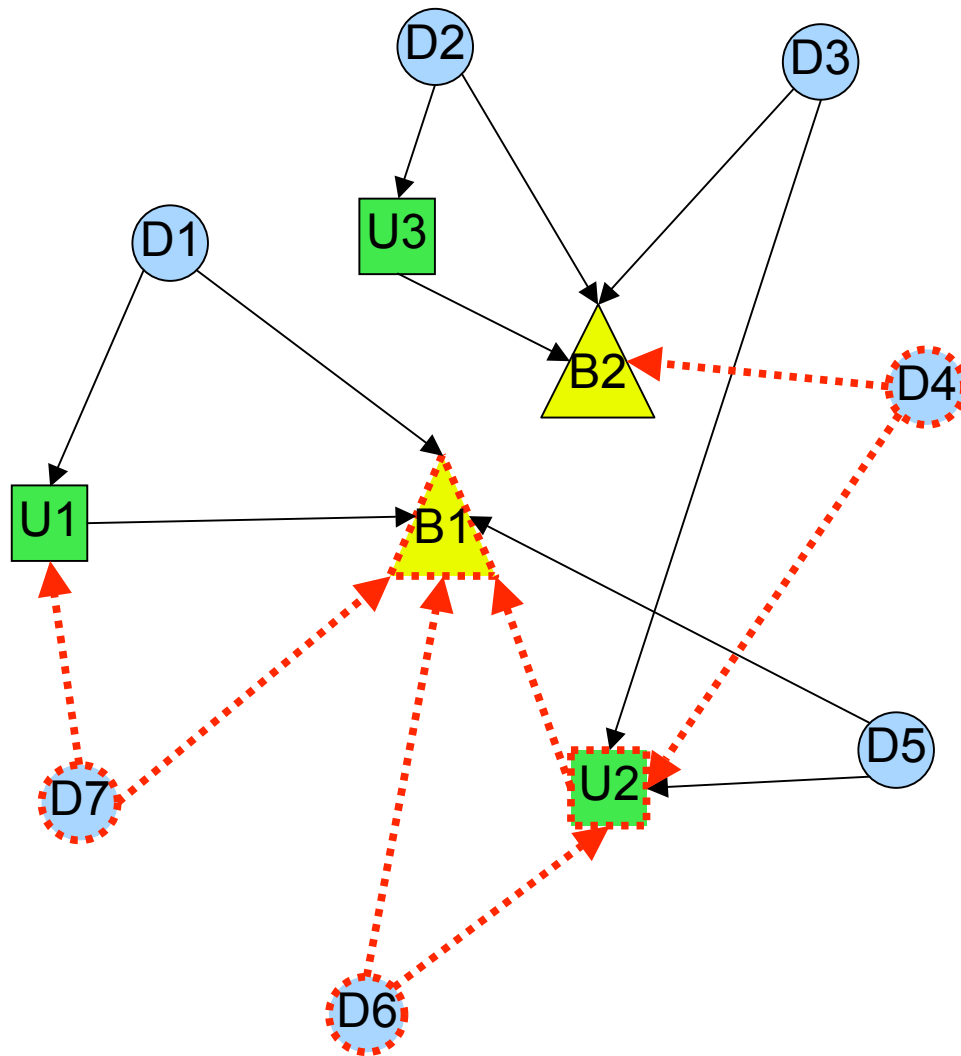


Figure 4: Degree distribution of the network: DS and US firms vs. banks

- The **degree distribution** of the interaction network tends to a power law
- The preferred-partner choice rule makes *preferential attachment* endogenous through a mechanism similar to the *fitness model*
- Economic behaviour, financial conditions and network evolution: **financially robust lenders** can supply credit at better conditions and therefore increase their market share. The opposite holds true for financially fragile agents. As a consequence, the corporate and the banking sectors become **polarized** and the degree distribution becomes **asymmetric**.
- **Robustness**: the network is robust to random shock.
- **Vulnerability**: the network is vulnerable to targeted shocks, because the default of a highly connected agent (rare event) may produce other defaults...



A typical story:

- D4, D6 and D7 go bankrupt due to idiosyncratic shocks
- They do not fulfill debt commitments
- The financial conditions of lenders deteriorate due to bad loans...
- In this case, U2 and B1 go bankrupt, while U1 and B2 survive to the failure of their partners
- Channel of bankruptcy propagation:
 - The failure of D4 and D6 provokes the default of U2
 - The failure of D6, D7 and, in particular, of U2 provokes the default of B1
- The deterioration of the financial conditions of U1 and B2 may produce an increase of the interest rate...
- The high rate of bankruptcy is a cause of the high interest rate as much as a consequence of it!

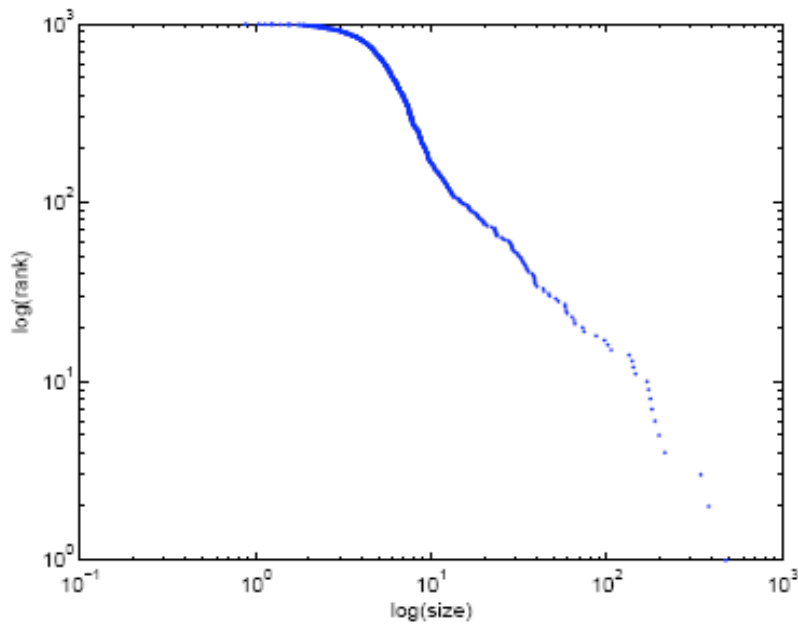


Figure 5: Bad debt distribution

- The extent of bankruptcy depends on the amount of **bad debt**
- The deterioration of lenders' financial condition due to borrowers' bankruptcies may be absorbed if the size of the non-performing loans is “small enough” or the lenders' net worth is “high enough”

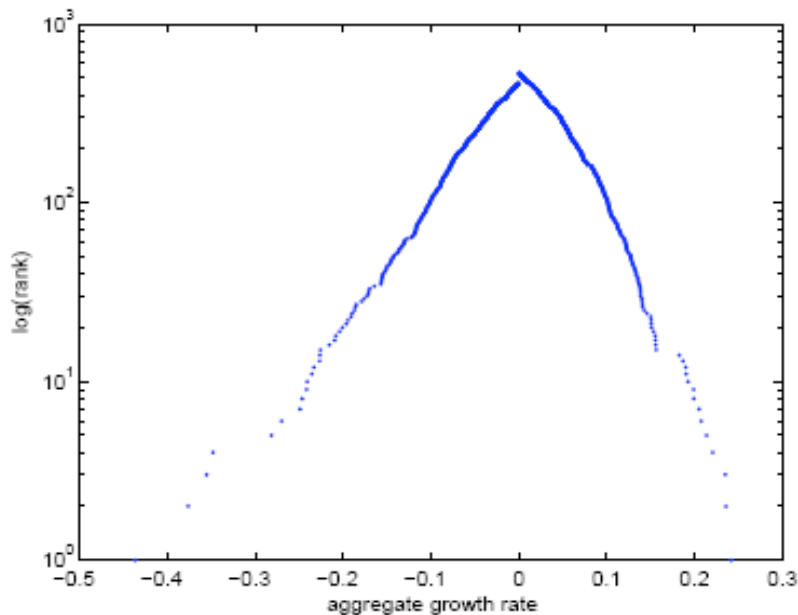


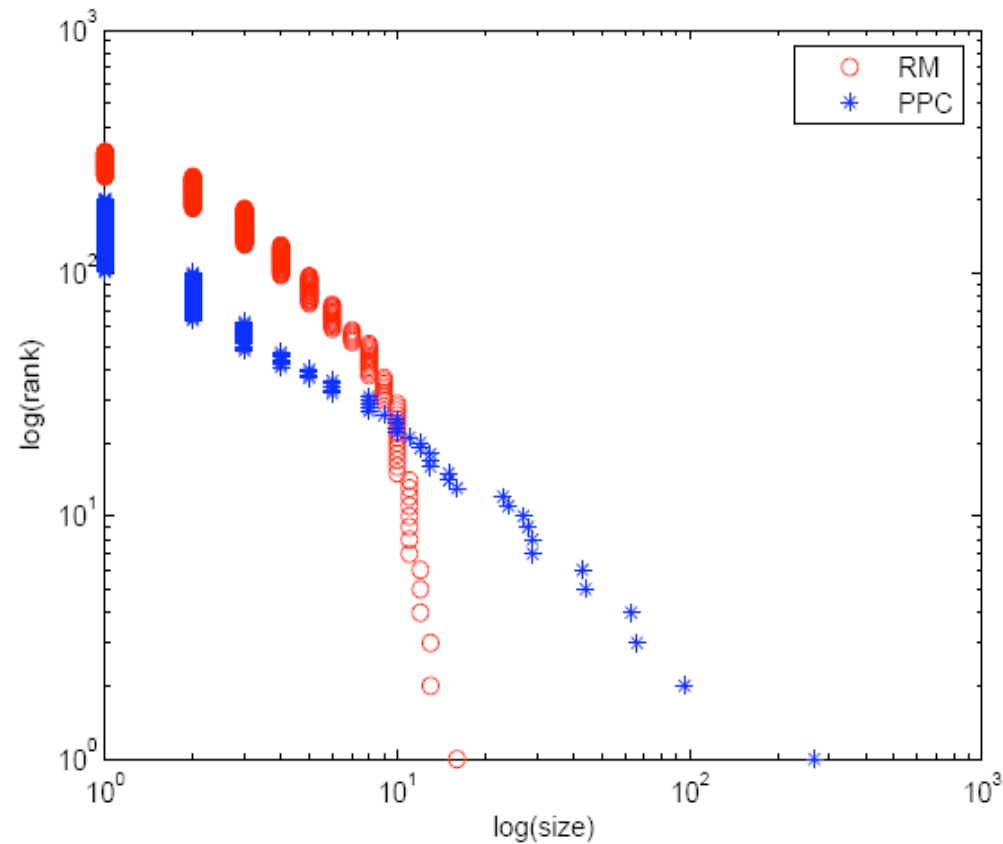
Figure 6: The distribution of aggregate growth rates

- The distribution of aggregate **growth rates** is far from being Gaussian (tent-shaped or double exponential)
- Asymmetry for **negative events**

PPC vs. RM

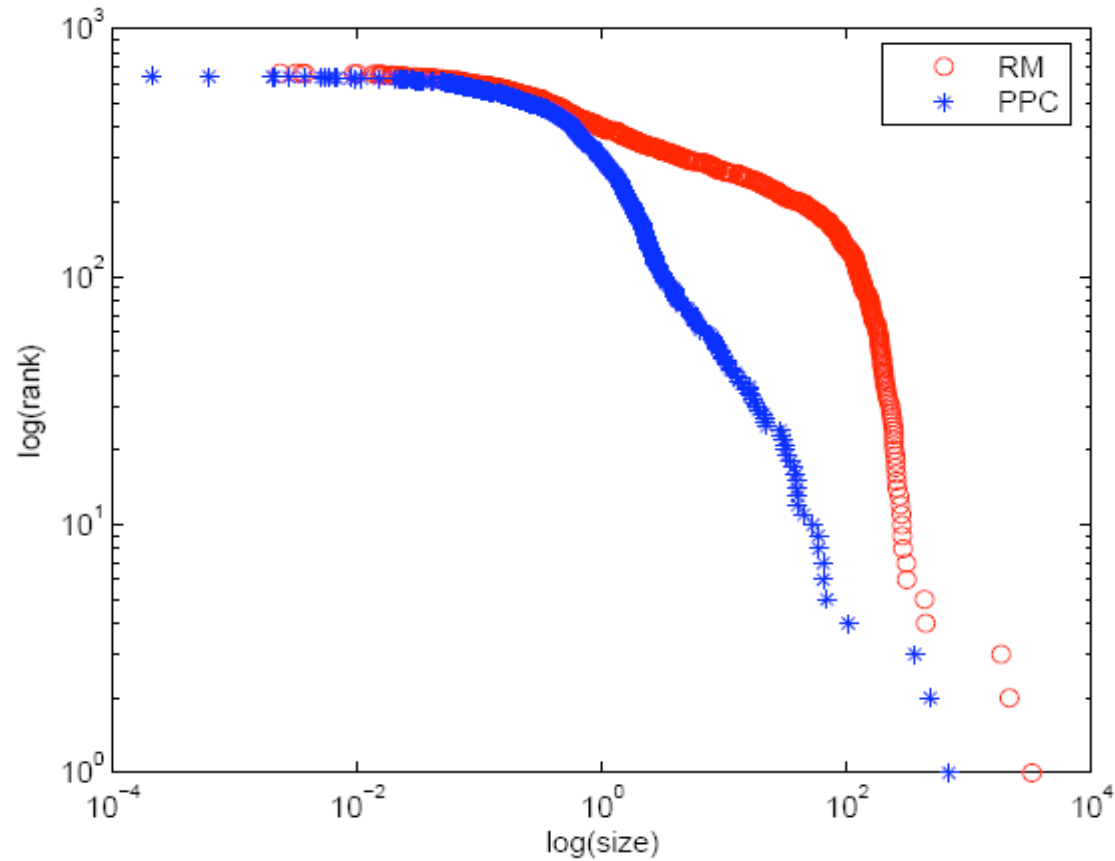
- Random network:
 - in every period each agent chooses a partner at random (model simulation with unchanged parameters but for $\varepsilon = 1$)
- Major findings:
 - Degree distribution of the network (credit interlinkages)
 - Firm size distribution (agents' heterogeneity)
 - Bankruptcy propagation (correlation structure across sectors)
 - Bankruptcy probability (systemic risk)

PPC vs RM



- Degree distribution of the network
 - with PPC, the distribution of links tends to a power law

PPC vs RM



- Firm size distribution
 - with PPC, FSD tends to a power law shape

Table 3. Monte Carlo simulations: RM vs. PPC.

	<i>RM</i>	<i>PPC</i>
mean(GR)	0.0013 (0.0000)	0.0013 (0.0000)
standard deviation(GR)	0.0393 (0.0018)	0.0401 (0.0025)
skewness(GR)	-0.1238 (0.0838)	-0.1393 (0.0696)
kurtosis(GR)	2.9386 (0.1510)	2.9132 (0.1657)
median(GR)	0.0022 (0.0010)	0.0025 (0.0010)
median absolute deviation(GR)	0.0266 (0.0014)	0.0273 (0.0020)
robust skewness(GR)	-0.0279 (0.0315)	-0.0356 (0.0301)
robust kurtosis(GR)	0.0031 (0.0521)	-0.0018 (0.0567)
median(BD)	37.2940 (1.5381)	36.8541 (3.8340)
median absolute deviation(BD)	12.0540 (1.2254)	16.49 (2.7380)
robust skewness(BD)	0.6612 (0.0681)	0.6518 (0.0880)
robust kurtosis(BD)	0.4279 (0.1523)	0.4031 (0.1918)
bankruptcy rate: corr(D,U)	0.2683 (0.0327)	0.2895 (0.0261)
bankruptcy rate: corr(D,B)	0.2044 (0.0324)	0.2196 (0.0295)
bankruptcy rate: corr(U,B)	0.7970 (0.0240)	0.7472 (0.0194)
bankruptcy rate: corr(D+U,B)	0.4320 (0.0327)	0.4200 (0.0284)
bankruptcy probability	0.1009 (0.0000)	0.1093 (0.0011)
bankruptcy probability: D	0.1656 (0.0000)	0.1670 (0.0000)
bankruptcy probability: U	0.0104 (0.0000)	0.0253 (0.0014)
bankruptcy probability: B	0.0035 (0.0000)	0.0310 (0.0022)
corr(Debt-to-Equity,Y)	0.6024 (0.1540)	-0.0453 (0.2187)

- **Bankruptcy rate:** Correlations of bankruptcy rates are similar in RM vs. PPC
- **Bankruptcy probability:** The bankruptcy probability of U firms and banks is higher in PPC than in RM
- **Systemic risk:** even though the correlation among bankruptcies is similar in the two scenarios, the greater incidence of defaults in the U and banking sectors means that the endogenous network increases the likelihood of bankruptcy propagation, starting from idiosyncratic shocks regarding D firms

Concluding remarks

- Modelling of productive and credit interlinkages: Endogenous network formation through preferred-partner choice
- Credit relationships (network structure), bankruptcy propagation, business fluctuations: bankruptcy rate \leftrightarrow interest rate
- Skew distributions: Firm size distribution, degree distribution of networks, bad debt, negative asymmetry for growth rates, etc.
- Endogenous network (PPC) vs. random matching (RM): systemic risk
- Empirical analysis: validating simulation results
- Towards a “complete” credit-network economy
 - Remove the hypothesis of (exogenous) stochastic price
 - Remove the hypothesis of (exogenous) constant number of agents
 - Remove the hypothesis of “on demand” production of U firms
 - Introduce the interbank market to investigate monetary policy issues